

The New Planck Scale: Quantized Spin and Charge Coupled to Gravity

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Abstract

In the standard approach to defining a Planck scale where gravity is brought into the quantum domain, the Schwarzschild gravitational radius is set equal to the Compton wavelength. However, ignored thereby are the charge and spin, the fundamental quantized aspects of matter. The gravitational and null-surface radii of the Kerr-Newman metric are used to introduce spin and charge into a new extended Planck scale. The fine structure constant appears in the extended Planck mass and the recent discovery of the α variation with the evolution of the universe adds further significance. An extended Planck charge and Planck spin are derived. There is an intriguing suggestion of a connection with the α value governing high-energy radiation in Z-boson production and decay.

Traditionally, one derives the Planck mass by equating the gravitational radius $2Gm/c^2$ of a Schwarzschild mass with its Compton wavelength \hbar/mc . The body has no spin and no charge yet spin and charge are the fundamental quantized aspects of

matter. Since the Planck scale is to reflect the *quantized* union of gravity with matter, surely spin and charge should be incorporated. Fortunately, we have the important couplings of spin and charge to gravity in the form of the Kerr-Newman [1] metric. The gravitational radius corresponds now to the upper sign in

$$r_{\pm} = \frac{G}{c^2} \left(m \pm \sqrt{m^2 - \frac{q^2}{G} - \frac{c^2}{G^2} a^2} \right) \quad (1)$$

and the second radius with the lower sign is the equally interesting “null surface” radius. To connect with the quantum domain, we quantize the charge in units of the charge e of the electron and the angular momentum in units of the fundamental quantum of angular momentum \hbar , with respective quantum numbers N and s :

$$q = N e , \quad a = s \frac{\hbar}{m} . \quad (2)$$

(Note that the m appears again through the spin.) Setting the Kerr-Newman event horizon and null surface (eq. (1)) radii of the particles equal to their Compton wavelengths, and substituting the quantized charge and spin from eq. (2), we have

$$\frac{\hbar}{mc} = \frac{G}{c^2} \left(m \pm \sqrt{m^2 - \frac{N^2 e^2}{G} - \frac{c^2 \hbar^2 s^2}{G^2 m^2}} \right) . \quad (3)$$

Solving for m , we find that the mass which we now refer to as the *extended Planck mass* m_{plex} is

$$m_{plex} = m_{pl} \sqrt{\frac{2(1 + s^2)}{2 - \alpha N^2}} , \quad (4)$$

for both cases, where $\alpha \equiv e^2/\hbar c \simeq 1/137$ is the fine structure constant and m_{pl} designates the standard Planck mass $\sqrt{c\hbar/2G}$. From the extended Planck mass, the new Planck length and Planck time, i.e. the complete new Planck scale is found in the usual manner.

By eq. (4), the presence of either spin or charge leads to an increase in the value of m_{plex} as compared to the traditional m_{pl} . It is also interesting to find that these two fundamental quantities of physics, the (now extended) Planck scale and the fine structure constant, are actually linked. Moreover, the presence of the fine structure constant in eq. (4) provides an additional source of interest, given the current focus upon its apparent slow variation in time [2]-[5]. Following recent claims [2]-[5] that the value of the fine structure constant underwent changes during the last half of the history of the universe, we focus on the possibility that α could have had a considerably different value in the still more distant past. If α undergoes significant variations, then m_{plex} does as well. Although rather unorthodox in the low-energy regime, this idea appears quite naturally in the context of renormalization, in which the coupling “constants” are actually running couplings. In the standard model, the early universe expands and cools precipitously in its very first instants when it emerges from the big bang, and the energy scale drops substantially, allowing for significant variations in the values of the running couplings.

If the fine structure “constant” changes at all, a change in either c or e could be responsible - see Refs. [6] for a debate on the two possibilities. A time-varying α can be accommodated in the context of varying speed of light cosmologies, of which many proposals have appeared recently [7]-[13]. While the reported variation of α over the last 10^{10} years is minute (of the order of 10^{-5} [2]-[5]) and the variation of fundamental constants is restricted by primordial nucleosynthesis, it is quite conceivable that more radical changes could have occurred earlier in the history of the universe. Although the current evidence points to a small increase in α as we go forward in time over the time scale thus far surveyed, the essential point is that there is variation and this variation

could have been one of decrease from a larger value at a still earlier time.

To fix our ideas, suppose that $N = 5$ and s is of order unity. Then, if at sometime in the past, α assumed a value close to $8 \cdot 10^{-2}$ (approximately one order of magnitude larger than its present value), the value of the extended Planck mass m_{plex} would have been many orders of magnitude larger than its present-day value, regardless of the value of the quantum number s (larger values of N lead to large effects for smaller variations of α).

Extremal values are generally useful to gain insight and hence we note that the critical upper-limit N value in eq. (4) is $N = 16$ for the present α value of $1/137.036$. With this N value, the extended Planck mass becomes infinite for an α value of $1/128$. Interestingly, the α value governing high-energy radiation in Z-boson production and decay has been measured to be $1/127.934$. The Z-boson is electrically neutral. Could it be that the connection to the *extremal* value reflects this neutrality? Recalling the history of theorizing about the number 137, we see in this that there really may be some connection between fundamental constants and integers.

It is to be noted that the scope for the extension of the Planck scale is severely limited if one were to be restricted to the choice of the event horizon radius r_+ as opposed to the null surface radius r_- . From eq. (3) with the positive sign in front of the square root, we find the inequality

$$\frac{\hbar}{mc} - \frac{Gm}{c^2} \geq 0 \quad (5)$$

and hence, with eq. (4)

$$m_{pl} \leq m_{plex} \leq \sqrt{2} m_{pl} \quad (6)$$

These conditions in conjunction with eq. (4) place the following restrictions on the

allowed spin and charge quanta:

$$s^2 + N^2\alpha \leq 1, \quad N^2\alpha < 2, \quad (7)$$

and they lead to a spectrum of spin/charge values. The allowed values of s and N for $\alpha = 1/137$ are

- a) for $s = 0$, $N \leq 11$
- b) for $s = 1/2$, $N \leq 10$
- c) for $s = 1$, $N = 0$.

Spin-two is not allowed in this case which might evoke some surprise as the graviton is seen as a spin-two boson. However the extended Planck mass, as the traditional Planck mass, is very large whereas the graviton mass is zero to a very high level of accuracy ($m_{graviton} < 10^{-59}$ g). These are very different concepts.

Given the new extended approach, it is natural to introduce an extended *Planck charge* and a *Planck spin*. These quantities could be defined by assuming that the “Planck particle” considered is an extremal black hole, i.e. one defined by

$$m^2 = \frac{q^2}{G} + \frac{c^2}{G^2} a^2 \quad (8)$$

(corresponding to the equality in (7)) that is maximally charged ($s = 0$, $q = q_{max}$) or maximally rotating ($q = 0$, $s = s_{max}$). These requirements yield the extended Planck quantities

$$q_{plex} = \frac{e}{\sqrt{\alpha}} \simeq 11.7 e, \quad s_{plex} = 1 \quad (9)$$

(corresponding to the Planck angular momentum $L_{plex} = \hbar$ and now allowing for non-integral N). While q_{plex} is large but not extraordinarily so, L_{plex} is rather ordinary on

the scale of particles familiar at an energy much lower than the Planck scale.

According to the third law of black hole thermodynamics, an extremal black hole corresponds to zero absolute temperature, and is an unattainable state. If the third law survives in the Planck regime, the values of N and s are even further restricted, and the first of (7) should read as a strict inequality.

If one considers instead the null surface of radius r_- defined by eq. (1) and with (4), the inequalities

$$s^2 + N^2\alpha \geq 1, \quad N^2\alpha < 2 \quad (10)$$

follow.

In this case, the allowed values of s and N for $\alpha = 1/137$ are,

- a) for $s = 0$, $12 \leq N \leq 16$
- b) for $s = 1/2$, $11 \leq N \leq 16$
- c) for $s = 1$, $0 \leq N \leq 16$
- d) for $s = 2$, $0 \leq N \leq 16$

In this case, spin two is readily allowed and with the extremal $N = 16$ value, the α value of $1/128$ gives an infinite m_{plex}

Finally, a comment is in order regarding the frequently mentioned observation that the natural length scale of “grand unification”, the merging of the strong and the electroweak interactions, is only a few orders of magnitude larger than the standard Planck length scale. Thus, the suggestion arises that ultimately, gravitation may hold the key to a final “super-grand unification”, the unification of all the interactions. It must be remarked that any spin or α modification in the new Planck scale can only increase the mass scale and hence lower the length scale. The possibility to be faced is that

gravitation may remain disjoint from the other interactions in nature.

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